

Supernova-driven outflows and chemical evolution of dwarf spheroidal galaxies

Yong-Zhong Qian^{a,1} and G. J. Wasserburg^{b,1}

^aSchool of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455; and ^bThe Lunatic Asylum, Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125

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We present a general phenomenological model for the metallicity distribution (MD) in terms of [Fe/H] for dwarf spheroidal galaxies (dSphs). These galaxies appear to have stopped accreting gas from the intergalactic medium and are fossilized systems with their stars undergoing slow internal evolution. For a wide variety of infall histories of unprocessed baryonic matter to feed star formation, most of the observed MDs can be well described by our model. The key requirement is that the fraction of the gas mass lost by supernova-driven outflows is close to unity. This model also predicts a relationship between the total stellar mass and the mean metallicity for dSphs in accord with properties of their dark matter halos. The model further predicts as a natural consequence that the abundance ratios [E/Fe] for elements such as O, Mg, and Si decrease for stellar populations at the higher end of the [Fe/H] range in a dSph. We show that, for infall rates far below the net rate of gas loss to star formation and outflows, the MD in our model is very sharply peaked at one [Fe/H] value, similar to what is observed in most globular clusters. This result suggests that globular clusters may be end members of the same family as dSphs.

In this paper, we show that supernova-driven gas outflows play a prominent role in the chemical evolution of dwarf spheroidal galaxies (dSphs). In the framework of hierarchical structure formation based on the cold dark matter cosmology, dwarf galaxies are the building blocks of large galaxies such as the Milky Way. In support of this picture, some recent observations showed that elemental abundances in dSphs of the Local Group match those in the Milky Way halo at low metallicities (e.g., refs. 1–5; see ref. 6 for a review of earlier works). It is expected that detailed studies of chemical evolution of dwarf galaxies can shed important light on the formation and evolution of the Milky Way in particular and large galaxies in general. Here we present an analysis of the evolution of $[\text{Fe}/\text{H}] = \log(\text{Fe}/\text{H}) - \log(\text{Fe}/\text{H})_\odot$ focusing on dSphs. The approach is a phenomenological one that takes into account infall of gas into the dark matter halos associated with these galaxies, star formation (SF) within the accumulated gas, and outflows driven by supernova (SN) explosions. The sources for production of Fe are core-collapse SNe (CCSNe) from progenitors of 8–100 M_\odot and Type Ia SNe (SNe Ia) associated with stars of lower masses in binaries. Observations require that some SNe Ia must form early along with CCSNe without a significant delay. It will be shown that there is a direct and simple connection between the metallicity distribution (MD) for a given dSph and two parameters $\lambda_{\text{Fe}}/\lambda$ and α . The ratio refers to the net rate λ_{Fe} of Fe production and the net rate λ of gas loss to SF and SN-driven outflows, and α indicates the promptness for reaching peak infall rates. This model explicitly predicts the ratio of the stellar mass in the dSph to the total mass of the host dark matter halo.

Phenomenological models for chemical evolution have a long history (e.g., ref. 7) and were applied to dSphs previously (e.g., refs. 8–10). Dynamic models for dSphs including dark matter were also studied (e.g., refs. 11 and 12). The first effort was made in ref. 13 to reconcile models of hierarchical structure formation involving dark matter halos with the then-available luminosity-radius-metallicity relationships for dwarf galaxies. There it was shown that SN-driven outflows could explain the observed trends.

Recent observations (14) give results for eight dSphs with rather detailed structure of their MDs and provide a basis for exploring models of their chemical evolution (e.g., ref. 15). Among the key issues that we try to address are MDs exhibited by stellar populations of dSphs. Our general approach follows that of Lynden-Bell described in his incisive and excellent article on “theories” of the chemical evolution of galaxies (16). It will be shown that the metallicity at which the MD peaks is directly related to the efficiency of SN-driven outflows and that the MD of a dSph and the relationship between the stellar mass and the mean metallicity for these galaxies are direct consequences of the model. These results are in strong support of those of ref. 17, where earlier and less precise data on metallicities of dwarf galaxies were used to address this problem.

In our approach, we consider evolution of Fe in a homogeneous system of condensed gas governed by

$$\frac{dM_g}{dt} = \left(\frac{dM_g}{dt}\right)_{\text{in}} - \psi(t) - F_{\text{out}}(t), \quad [1]$$

$$\frac{dM_{\text{Fe}}}{dt} = P_{\text{Fe}}(t) - \frac{M_{\text{Fe}}(t)}{M_g(t)}[\psi(t) + F_{\text{out}}(t)], \quad [2]$$

where $M_g(t)$ is the mass of gas in the system at time t , $(dM_g/dt)_{\text{in}}$ is the infall rate of pristine gas, $\psi(t)$ is the star formation rate (SFR), $F_{\text{out}}(t)$ is the rate of gas outflow, $M_{\text{Fe}}(t)$ is the mass of Fe in the gas, and $P_{\text{Fe}}(t)$ is the net rate of Fe production by all sources in the system. We assume that the SFR is proportional to the mass of gas in the system with an astration rate constant λ_* ,

$$\psi(t) = \lambda_* M_g(t). \quad [3]$$

Given $(dM_g/dt)_{\text{in}}$, $F_{\text{out}}(t)$, and $P_{\text{Fe}}(t)$, Eqs. 1 and 2 can be solved with the initial conditions $M_g(0) = 0$ and $M_{\text{Fe}}(0) = 0$.

The MD of a system measures the numbers of stars formed in different metallicity intervals that survive until the present time. We use $[\text{Fe}/\text{H}]$ to measure metallicity. As the mass fraction of H changes very little over the history of the universe, we take $[\text{Fe}/\text{H}] = \log Z_{\text{Fe}}$, where

$$Z_{\text{Fe}}(t) \equiv \frac{M_{\text{Fe}}(t)}{X_{\text{Fe}}^\odot M_g(t)}. \quad [4]$$

Here X_{Fe}^\odot is the mass fraction of Fe in the sun. We assume that the initial mass function of SF does not change with time and is of the Salpeter form. Then the number of stars formed per unit mass interval per unit time is related to the SFR as

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¹To whom correspondence may be addressed. E-mail: qian@physics.umn.edu or gjw@gps.caltech.edu.

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$$\frac{d^2 N}{dm dt} = \left[\frac{\psi(t)}{M_\odot} \right] \frac{m^{-2.35}}{\int_{m_l}^{m_u} m^{-1.35} dm}, \quad [5]$$

where m is the stellar mass in units of M_\odot with m_l and m_u being the lower and upper limits, respectively. We take $m_l = 0.1$ and $m_u = 100$. Assuming that $Z_{\text{Fe}}(t)$ increases monotonically with time, we obtain the MD

$$\begin{aligned} \frac{dN}{d[\text{Fe}/\text{H}]} &= \frac{\int_{0.1}^{m_{\text{max}}(t)} (d^2 N / dm dt) dm}{d[\text{Fe}/\text{H}] / dt} \\ &= \left(\frac{\lambda_*}{\log e} \right) \frac{\int_{0.1}^{m_{\text{max}}(t)} m^{-2.35} dm}{\int_{0.1}^{100} m^{-1.35} dm} \\ &\quad \times \left[\frac{M_g(t)}{M_\odot} \right] \frac{Z_{\text{Fe}}(t)}{dZ_{\text{Fe}}/dt}, \end{aligned} \quad [6]$$

where $m_{\text{max}}(t)$ is the maximum mass of those stars formed at time t that survive until the present time (cf. ref. 18). There is little SF in dSphs at the present time. We assume that SF ended at time t_f in a system. Then the total number of stars in the system at the present time is

$$\begin{aligned} N_{\text{tot}} &= \int_0^{t_f} \int_{0.1}^{m_{\text{max}}(t)} \frac{d^2 N}{dm dt} dm dt \\ &= \frac{\lambda_*}{\int_{0.1}^{100} m^{-1.35} dm} \int_0^{t_f} \frac{M_g(t)}{M_\odot} \int_{0.1}^{m_{\text{max}}(t)} \frac{dm dt}{m^{2.35}}. \end{aligned} \quad [7]$$

The integral involving $m_{\text{max}}(t)$ in Eqs. 6 and 7 increases only by 6% when $m_{\text{max}}(t)$ increases from 0.8 to 100. Because stars with $m = 0.8$ have a lifetime approximately equal to the age of the universe, we take $m_{\text{max}}(t) = 0.8$ in both these equations to obtain the normalized MD

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{1}{\log e} \left[\frac{M_g(t)}{\int_0^{t_f} M_g(t) dt} \right] \frac{Z_{\text{Fe}}(t)}{dZ_{\text{Fe}}/dt}. \quad [8]$$

Model

The key input for our model is the infall rate $(dM_g/dt)_{\text{in}}$, the outflow rate $F_{\text{out}}(t)$, and the net Fe production rate $P_{\text{Fe}}(t)$. The latter two rates are closely related to the occurrences of CCSNe and SNe Ia in a system. CCSNe are associated with massive stars ($8 < m \leq 100$) that evolve rapidly. In contrast, SNe Ia require consideration of the evolution of binaries involving lower-mass stars with longer lifetimes. In all our previous studies (e.g., ref. 19), we considered that the evolution timescale for SNe Ia was ~ 1 Gy using the lifetime of stars with $m \sim 2$. This consideration meant that SNe Ia would not contribute to the Fe inventory during early epochs and was in accord with the general approach used by other workers. The consequence of this assumption is that the MD for a system must have two peaks due to the assumed late onset of SNe Ia (e.g., ref. 19). However, of the eight dSphs studied in ref. 14, only a single peak is observed in the MD for Fornax, Leo I, Leo II, Sextans, Draco, and Canes Venatici I, and there is only some indication for two peaks in the MD for Ursa Minor and perhaps Sculptor. From this observation, we conclude that there must be a prompt component of SNe Ia that start in a system on much shorter timescales than approximately 1 Gy. Such a component is supported by both supernova surveys (e.g., ref. 20) and models that considered detailed evolution of various binary configurations (e.g., refs. 21 and 22). The occurrences of “prompt” and “delayed” SNe Ia were investigated in ref. 23, where it was argued that both populations were present at high redshift. The nature of these two classes of SNe Ia remains unclear. For simplicity, we will lump the Fe production by these sources together with that by CCSNe and ignore the time delay

between the birth and death of all SN progenitors, except when the amount of gas in the system is so low that SNe Ia would dominate.

Under our assumption, the total rate of CCSNe and SNe Ia, and hence $P_{\text{Fe}}(t)$, are proportional to the SFR. We further assume that the rate of outflows driven by these SNe is also proportional to the SFR. Specifically, we take

$$F_{\text{out}}(t) = \eta \lambda_* M_g(t), \quad [9]$$

$$P_{\text{Fe}}(t) = \lambda_{\text{Fe}} X_{\text{Fe}}^\odot M_g(t), \quad [10]$$

where η is a dimensionless constant that measures the efficiency of the SN-driven outflows, and λ_{Fe} is a rate constant that is proportional to λ_* and the effective Fe yield of SNe. We take the infall rate to be

$$\left(\frac{dM_g}{dt} \right)_{\text{in}} = \lambda_{\text{in}} M_0 \frac{(\lambda_{\text{in}} t)^\alpha}{\Gamma(\alpha + 1)} \exp(-\lambda_{\text{in}} t), \quad [11]$$

where λ_{in} is a rate constant, M_0 is the total mass of gas infall over $0 \leq t < \infty$, and $\Gamma(\alpha + 1)$ with $\alpha > -1$ is the Γ function of argument $\alpha + 1$. The above modified exponential form was specifically chosen to explore the role of the time dependence of the infall rate in chemical evolution (cf. ref. 7). The infall rate peaks at $t = 0$ for $-1 < \alpha \leq 0$, and the peak time increases to $\alpha/\lambda_{\text{in}}$ for $\alpha > 0$. The form with $\alpha > 0$ allows a slow start of significant gas accumulation in the system.

With the above assumptions, Eqs. 1 and 2 become

$$\frac{dM_g}{dt} = \lambda_{\text{in}} M_0 \frac{(\lambda_{\text{in}} t)^\alpha}{\Gamma(\alpha + 1)} \exp(-\lambda_{\text{in}} t) - \lambda M_g(t), \quad [12]$$

$$\frac{dM_{\text{Fe}}}{dt} = \lambda_{\text{Fe}} X_{\text{Fe}}^\odot M_g(t) - \lambda M_{\text{Fe}}(t), \quad [13]$$

where $\lambda \equiv (1 + \eta)\lambda_*$. Note that, for $\lambda_{\text{in}} \ll \lambda$, the solutions to the above equations approach a secular state for which $M_g(t) \approx (dM_g/dt)_{\text{in}}/\lambda$ and $Z_{\text{Fe}} \approx \lambda_{\text{Fe}}/\lambda$. This situation is analogous to the quasi-steady state for the metallicity of the interstellar medium first proposed in ref. 24. For simplicity, we assume that λ_{in} and λ are so large that $t_f = \infty$ can be used effectively in Eq. 8 for the MD. As $M_g(0) = M_g(\infty) = 0$, integrating Eq. 12 over t gives $\lambda \int_0^\infty M_g(t) dt = M_0$. So the total gas mass used in SF is $\lambda \int_0^\infty M_g(t) dt = M_0/(1 + \eta)$ and the remainder of the gas infall is blown out as outflows. We assume that all outflows are lost from the system into the broader intergalactic medium (IGM), thus enriching the latter in metals. This approach gives a natural cutoff to the chemical evolution of the system when the total mass of gas lost to SF and outflows is equal to the total mass of gas infall (partial recycling of outflows would increase the degree of chemical enrichment but is ignored here for simplicity). At the present time, the total mass of stars in the system can be estimated as

$$M_* = \frac{\int_{0.1}^{0.8} m^{-1.35} dm}{\int_{0.1}^{100} m^{-1.35} dm} \frac{M_0}{1 + \eta} = 9.65 \times 10^{-2} \left(\frac{M_h}{1 + \eta} \right), \quad [14]$$

where we have used $M_0 = (\Omega_b/\Omega_m)M_h = 0.17M_h$. Here M_h is the total mass inside the dark matter halo hosting the system, and Ω_b and Ω_m are the fractional contributions to the critical density of the universe from baryonic and all matter, respectively.

Exploration of Results from the Model

The parameters governing the solutions to Eqs. 12 and 13 are α , λ_{in} , λ , and λ_{Fe} . We illustrate the dependences of the MD on these parameters in the following subsections.

Dependence of the MD on $\lambda_{\text{in}}/\lambda$ with $\alpha = 0$. For $\alpha = 0$, the solutions to Eqs. 12 and 13 are

$$M_g(t) = \frac{\lambda_{\text{in}}}{\lambda_{\text{in}} - \lambda} M_0 [\exp(-\lambda t) - \exp(-\lambda_{\text{in}} t)], \quad [15]$$

$$M_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} \lambda_{\text{in}}}{(\lambda_{\text{in}} - \lambda)^2} X_{\text{Fe}}^{\odot} M_0 \{ \exp(-\lambda_{\text{in}} t) + [(\lambda_{\text{in}} - \lambda)t - 1] \exp(-\lambda t) \}, \quad [16]$$

from which we obtain

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}}}{\lambda_{\text{in}} - \lambda} \left[\frac{(\lambda_{\text{in}} - \lambda)t}{1 - \exp[-(\lambda_{\text{in}} - \lambda)t]} - 1 \right], \quad [17]$$

$$\begin{aligned} \frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} &= \frac{1}{\log e} \frac{\lambda_{\text{in}} \lambda}{(\lambda_{\text{in}} - \lambda)^2} \frac{\{1 - \exp[-(\lambda_{\text{in}} - \lambda)t]\}^2}{\exp(\lambda t)} \\ &\times \frac{(\lambda_{\text{in}} - \lambda)t - 1 + \exp[-(\lambda_{\text{in}} - \lambda)t]}{1 - [1 + (\lambda_{\text{in}} - \lambda)t] \exp[-(\lambda_{\text{in}} - \lambda)t]}. \end{aligned} \quad [18]$$

For $\lambda_{\text{in}}/\lambda = 1$, the above two equations reduce to

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} t}{2}, \quad [19]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{(\lambda t)^2}{\log e} \exp(-\lambda t). \quad [20]$$

In the limit $\lambda_{\text{in}} = \infty$, which is equivalent to setting $M_g(0) = M_0$ and $(dM_g/dt)_{\text{in}} = 0$ for $t > 0$, the results are

$$Z_{\text{Fe}}(t) = \lambda_{\text{Fe}} t, \quad [21]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{\lambda t}{\log e} \exp(-\lambda t). \quad [22]$$

In general, Eqs. 17 and 18 give the MD as a function of $[\text{Fe}/\text{H}]$ in parametric form. This MD only depends on $\lambda_{\text{in}}/\lambda$ and $\lambda_{\text{Fe}}/\lambda$, but not on the absolute values of these rates (this is true so long as $\lambda t_f \gg 1$ and $\lambda_{\text{in}} t_f \gg 1$; see Eqs. 8 and 15). For a fixed $\lambda_{\text{in}}/\lambda$, changing $\lambda_{\text{Fe}}/\lambda$ only translates the MD along the $[\text{Fe}/\text{H}]$ axis. This result can be most easily seen in the special cases of $\lambda_{\text{in}}/\lambda = 1$ and $\lambda_{\text{in}} = \infty$, where $N_{\text{tot}}^{-1} dN/d[\text{Fe}/\text{H}]$ is a simple function of λt and $[\text{Fe}/\text{H}]$ differs from $\log(\lambda t)$ only by a shift of $\log(\lambda_{\text{Fe}}/2\lambda)$ and $\log(\lambda_{\text{Fe}}/\lambda)$, respectively. The shape of the MD is determined by $\lambda_{\text{in}}/\lambda$. This dependence can be seen from Fig. 1A, which uses $\lambda_{\text{Fe}}/\lambda = 0.1$ and shows that, as $\lambda_{\text{in}}/\lambda$ increases from 1/2 to ∞ , the position of the peak of the MD changes slightly [but staying close to $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda) = -1$, the exact peak position for $\lambda_{\text{in}}/\lambda = 1$ and $\lambda_{\text{in}} = \infty$] and the shape of the MD becomes broader. Note the sharp cutoff of the MD to the right of the peak for $\lambda_{\text{in}}/\lambda = 1/2$ with no stars formed above $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})] = -0.7$.

The case of $\lambda_{\text{in}}/\lambda < 1/2$ requires separate discussion. For illustration, we again take $\lambda_{\text{Fe}}/\lambda = 0.1$ and show the MD for $\lambda_{\text{in}}/\lambda = 0.1$ in Fig. 1B. This MD has an extremely sharp peak with 90% of the stars having $-1.154 \leq [\text{Fe}/\text{H}] < -0.954$. This result is simply a case close to secular equilibrium (see discussion below Eqs. 12 and 13; cf. ref. 24). Piling up of stars in an extremely narrow

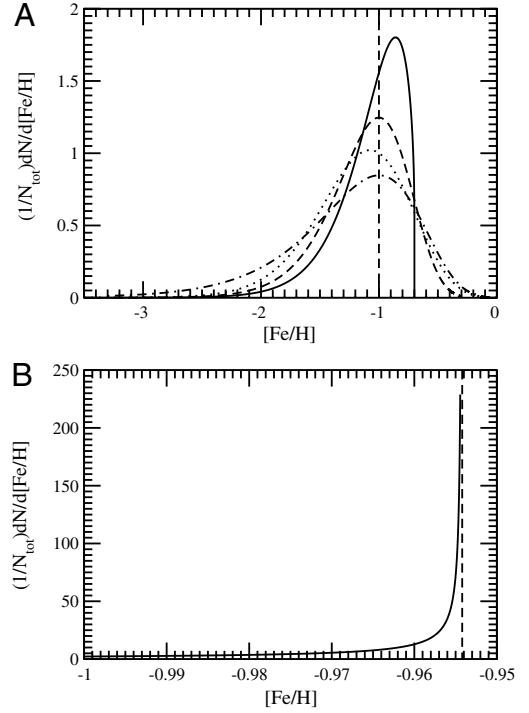


Fig. 1. Example MDs for $\lambda_{\text{Fe}}/\lambda = 0.1$ and various values of $\lambda_{\text{in}}/\lambda$ with $\alpha = 0$. Note that in general the shape of the MD is determined by $\lambda_{\text{in}}/\lambda$ and changing $\lambda_{\text{Fe}}/\lambda$ only translates the MD along the $[\text{Fe}/\text{H}]$ axis. (A) The solid, dashed, dotted, and dot-dashed curves are the MDs for $\lambda_{\text{in}}/\lambda = 1/2, 1, 2$, and ∞ , respectively. The vertical dashed line indicates the peak at $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda) = -1$ for the dashed and dot-dashed curves. (B) The solid curve is the MD for $\lambda_{\text{in}}/\lambda = 0.1$ and the dashed line indicates the limiting value of $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})] = -0.954$. Concentration of stars in an extremely narrow range of $[\text{Fe}/\text{H}]$ as shown by the solid curve is typically not observed for dSphs but strongly resembles what is observed for most globular clusters.

metallicity range is typically not observed for dwarf galaxies but strongly resembles what is observed for most globular clusters. In general, for $\lambda_{\text{in}}/\lambda$ significantly below 1/2, stars are concentrated immediately below $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})]$. For comparison, the solid curve in Fig. 1A shows that the MD for $\lambda_{\text{in}}/\lambda = 1/2$ first rises to a peak and then sharply drops to zero as $[\text{Fe}/\text{H}] \rightarrow \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})]$. This general behavior also applies to $\lambda_{\text{in}}/\lambda > 1/2$ but with a more extended tail at high metallicities for a larger $\lambda_{\text{in}}/\lambda$ (see Fig. 1A).

Dependence of the MD on α with $\lambda_{\text{in}} = \lambda$. Based on the above discussion and the example MDs shown in Fig. 1, it seems reasonable to choose $\lambda_{\text{in}} = \lambda$ and explore possible MDs for different values of α and $\lambda_{\text{Fe}}/\lambda$. For $\lambda_{\text{in}} = \lambda$ and $\alpha > -1$, the solutions to Eqs. 12 and 13 are

$$M_g(t) = M_0 \frac{(\lambda t)^{\alpha+1}}{\Gamma(\alpha+2)} \exp(-\lambda t), \quad [23]$$

$$M_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}}}{\lambda} X_{\text{Fe}}^{\odot} M_0 \frac{(\lambda t)^{\alpha+2}}{\Gamma(\alpha+3)} \exp(-\lambda t), \quad [24]$$

which give

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} t}{\alpha + 2}, \quad [25]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{1}{\log e} \frac{(\lambda t)^{\alpha+2}}{\Gamma(\alpha+2)} \exp(-\lambda t). \quad [26]$$

Sculptor with intermediate M_* . Using the same $\lambda_{\text{Fe}}/\lambda$, we obtained a better fit to the data for Leo II with $\lambda_{\text{in}}/\lambda = 0.8$ and $\alpha = 0$ (dashed curve in Fig. 2C) than with $\lambda_{\text{in}}/\lambda = 1$ and $\alpha = 1$ (solid curve). However, neither type of MD can provide a good fit to the data for Sculptor, which may represent an MD with two peaks. There is perhaps a more clear indication for such an MD for Ursa Minor. We note that, if infall took a more complicated form than Eq. 11, then the shape of the MD would change accordingly. It is possible that the broadened peak in the MD for Sculptor might be explained by an increase in the infall rate after the assumed smooth rate in Eq. 11 peaked. However, in no case can a significant part of the enriched outflows be returned to the infalling matter because this would produce a population of stars with high $[\text{Fe}/\text{H}]$ values, which is not observed.

In a previous study (14) with alternative modeling, detailed multiparameter fitting was used. As can be seen from Fig. 2, a good to excellent fit of the model to the data can be obtained in our approach. The model thus appears to give a good description of the MDs of dSphs and is easily understood in terms of physical processes.

Relationship Between $\langle[\text{Fe}/\text{H}]\rangle$ and M_* for dSphs. The observed MDs of dSphs require that $\lambda_{\text{Fe}}/\lambda \sim 0.01\text{--}0.1$ (see Fig. 2). Because $\lambda_{\text{Fe}}/\lambda$ is approximately $(1 + \eta)^{-1}$ (see derivation of λ_{Fe} in *SI Text*), this requires that $\eta \sim 10\text{--}100$. Consequently, only a fraction $(1 + \eta)^{-1} \sim 1\text{--}10\%$ of the gas falling into the dark matter halo hosting a dSph is used in SF and the rest is blown out of the halo by SN-driven outflows. The MD of a dSph peaks at essentially its mean metallicity $\langle[\text{Fe}/\text{H}]\rangle \approx \log(\lambda_{\text{Fe}}/\lambda) \sim -\log \eta$. The lower $\langle[\text{Fe}/\text{H}]\rangle$ a dSph has, the lower fraction of the infalling gas is stored in its stars. There is thus a direct relationship between $\langle[\text{Fe}/\text{H}]\rangle$ and the stellar mass M_* for dSphs. Using the data on $\langle[\text{Fe}/\text{H}]\rangle$ in ref. 14 and those on M_* in refs. 25 and 26, we show this relationship in Fig. 3. The dot-dashed line in Fig. 3 has a slope of 2.5 and passes through the point defined by the average values of the data. It is nearly the same as the least-square fit (solid line) and is in excellent agreement with the result in ref. 25 (see *SI Text*).

As discussed in ref. 17, the slope of 2.5 for the relationship between $\log(M_*/M_\odot)$ and $\langle[\text{Fe}/\text{H}]\rangle$ has a simple physical explanation. In general, this slope follows from the relationship between the radius r_h and the total mass M_h of the dark matter halo hosting a dSph in addition to the dependences of M_* and $\langle[\text{Fe}/\text{H}]\rangle$ on η discussed above. The outflow efficiency η is inversely proportional to the depth of the gravitational potential well of the dark matter halo,

$$\eta \propto r_h/M_h \propto M_h^{\beta-1}, \quad [27]$$

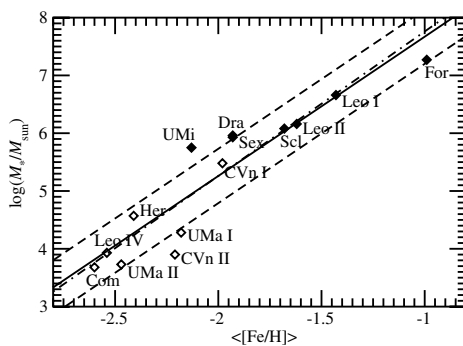


Fig. 3. The relationship between $\log(M_*/M_\odot)$ and $\langle[\text{Fe}/\text{H}]\rangle$ for dSphs. Values of M_* for filled and open diamonds are taken from refs. 25 and 26, respectively. All values of $\langle[\text{Fe}/\text{H}]\rangle$ used here are taken from ref. 14. The solid line is the least-square fit to the data and the dashed lines indicate the 1σ error in $\log(M_*/M_\odot)$. The dot-dashed line has a slope of 2.5 and is nearly the same as the solid line.

where we have assumed $r_h \propto M_h^\beta$ in the second step, which gives

$$\langle[\text{Fe}/\text{H}]\rangle \sim -\log \eta \sim (1 - \beta) \log M_h + \text{constant}. \quad [28]$$

In addition, Eq. 14 gives

$$M_* \propto M_h/\eta \propto M_h^{2-\beta}. \quad [29]$$

Thus, a slope of 2.5 for the relationship between $\log(M_*/M_\odot)$ and $\langle[\text{Fe}/\text{H}]\rangle$ requires $(2 - \beta)/(1 - \beta) = 2.5$, or $\beta = 1/3$, which is in agreement with the framework of hierarchical structure formation (e.g., Eq. 24 in ref. 27).

Conclusions

We have shown that the MDs of dSphs can be reasonably well described quantitatively by a phenomenological model based on general considerations of gas infall into a dark matter halo, astration in the condensed gas, and SN-driven gas outflows. The peak position of the MD is governed by the ratio of the rate constants for net Fe production and net gas loss to astration and outflows, $\lambda_{\text{Fe}}/\lambda = \lambda_{\text{Fe}}/[(1 + \eta)\lambda_*]$. The observations require high efficiency $\eta \sim 10\text{--}360$ (see Table S1) for SN-driven outflows and hence, massive gas loss from dark matter halos associated with dSphs. The model also directly relates the stellar mass M_* remaining in a dSph to its mean metallicity $\langle[\text{Fe}/\text{H}]\rangle$ through the efficiency of outflows governed by the mass M_h and the radius r_h of the dark matter halo. The observed relationship between $\log(M_*/M_\odot)$ and $\langle[\text{Fe}/\text{H}]\rangle$ has a slope of 2.5 over the wide range of $4.8 \times 10^3 \leq M_*/M_\odot \leq 4.6 \times 10^8$ for dwarf galaxies (see *SI Text*), indicating $r_h \propto M_h^{1/3}$ in agreement with the framework of hierarchical structure formation. Our results confirm the previous studies of refs. 17 and 25, and are in support of the early work of Dekel and Sulk (13) on the general model of dwarf galaxy formation.

Our model also demonstrates that, for slow infall rates $\lambda_{\text{in}} \ll \lambda$, the resulting MD must be extremely sharply peaked (essentially concentrated at a single value of $[\text{Fe}/\text{H}]$). Such MDs apply to all globular clusters with the exception of the most massive ones (e.g., ref. 28), suggesting that globular clusters are the result of the general process of astration governed by very slow infall rates compared to net gas loss rates. These slow infall rates further suggest that globular clusters might be weak feeders during the inhomogeneous evolution inside a large system and that they are not responsible for significant depletion of the baryonic supply of that system. As such, the dark matter halo would be associated with the entire (much larger) system, but not be specifically tied to the globular clusters. This behavior is in contrast to that of dSphs, which result from processing all the baryonic supply in their dark matter halos and are naturally associated with these dark matter halos. Nonetheless, from the point of view of the analysis presented here, we consider globular clusters to be part of a family of early formed dwarf galaxies but with low effective infall rates compared to outflow rates. With regard to the diverse morphological types of dwarf galaxies, we consider dSphs to represent isolated evolution without dynamic effects from mergers or tidal interactions with nearby systems. Other morphological types may result from such effects. Insofar as all dwarf galaxies are related by the same general process as presented here, the observed ongoing astration in some dwarf irregular galaxies (e.g., ref. 18) poses a problem. These systems must be experiencing secondary processes of gas accretion due to local infall or mergers. Their MDs should have a second peak due to the late infall.

The analysis presented here assumes that there is not a discontinuous onset of SNe Ia contributing Fe. Otherwise, the general outcome would be MDs with two peaks, which are at most only rarely observed. It is not appropriate to extend this analysis to other metals than Fe without a more realistic treatment of the relative contributions of CCSNe and SNe Ia that each produce

very different yields of the other metals. We note here that, as the rate of CCSNe decreases with decreasing gas mass, the contributions to Fe from SNe Ia will become larger or dominant. The reason is that previously formed stars of low to intermediate masses in binaries will continue to evolve and produce SNe Ia even after the SFR decreases. Crudely speaking, the net Fe production rate appropriate for later times must be changed from Eq. 10 to $P_{\text{Fe}}(t) \sim X_{\text{Fe}}^{\odot} [\lambda_{\text{Fe}}^{\text{CC}} M_g(t) + \lambda_{\text{Fe}}^{\text{Ia}} M_g(t - \Delta)]$, where $\lambda_{\text{Fe}}^{\text{CC}}$ and $\lambda_{\text{Fe}}^{\text{Ia}}$ are the rate constants for CCSNe and SNe Ia, respectively, and Δ is a typical delay between the birth and death of the progenitors for SNe Ia. As SNe Ia do not produce, e.g., O, Mg, and Si, this will result in lower values of $[E/\text{Fe}]$ for these elements at higher values of $[\text{Fe}/\text{H}]$, which is in agreement with the general trend observed in dSphs (e.g., ref. 15).

In conclusion, dSphs have evolved as a result of very massive gas loss and this gas has gone into the medium outside the dark matter halos associated with these galaxies. The implication is that these dark matter halos must have masses approximately 10^2 – 10^3 times greater than the present total mass in stars after the gas loss and the cosmic ratio of baryonic to all matter are taken into account (see Table S1). This conclusion is in qualitative

accord with the conclusions from previous studies to infer the halo masses from observations of dSphs (e.g., ref. 29). As a result of the massive gas loss, extensive enrichment of metals has occurred in the general hierarchical structures outside individual dSphs. This enrichment drastically alters the subsequent chemical evolution of the emerging larger galaxies as most baryonic matter must have passed through processing in dSphs. As more massive dark matter halos are formed during hierarchical growth, the efficiency of outflows will decrease. Nevertheless, the general IGM must have been enriched by the net outflows from all dSphs. If the average IGM has $[\text{Fe}/\text{H}] \sim -3$ and the outflows from dSphs have $[\text{Fe}/\text{H}] \sim -1.5$ on average, this would imply that approximately 3% of all baryonic matter was processed in dSphs.

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